Charge-odd and single-spin effects in two-pion production in $e\vec{p}$ collisions*

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Abstract

We consider double-photon and bremsstrahlung mechanisms for the production of two charged pions in high-energy electron (or proton) scattering off a transversely polarised proton. Interference between the relevant amplitudes generates a charge-odd contribution to the cross-section for the process. In the kinematical configuration with a jet nearly collinear to the electron, the spin-*independent* part may be used to the determine phase differences for pion-pion scattering in states with orbital momentum 0 or 2 and 1, while for the configuration with a jet nearly collinear to the proton, the spin-*dependent* part may be used to explain the experimental data for single-spin correlations in the production of negatively charged pions. We also discuss the backgrounds and estimate the accuracy of the results to be better than 10%. In addition, simplified formulæ derived for specific kinematics, with small total transverse pion momenta, are given.

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1 Introduction

Among the possible methods of measuring pion-pion scattering phases at low energies [1] (*e.g.*, K_{e4} decays, pionium atoms, pion-to-two-pion transitions in the scattering of pions by protons—the Chew-Low process), high-energy processes in which a jet consisting of pions with relative small invariant mass is created have not, up to now, been considered. The theoretical possibility was, however, considered some time ago by Serbo and Chernyak [2] for a special kinematical region of the produced pions.

In this paper we calculate the charge-odd contribution to the cross-section in charged-pion pair production for the kinematics of a jet moving close to the direction of one of the initial particles. We consider the general case, restricted only by the requirement that the sum of the momenta transverse to the beam axis be close to zero. This region corresponds to the case where one of the scattered particles moves close to its initial direction and escapes detection while the components of the jet moving in the opposite direction have measurable scattering angles and are assumed to be detected.

The bremsstrahlung mechanism of pion pair creation includes the conversion of a time-like photon into a pion pair via an intermediate ρ-meson state. The Breit-Wigner resonance form of the relevant pion form factor provides an imaginary part, which can give rise to single-spin correlation effects in the differential cross-section. These last may also arise as interference effects between the Born and one-loop Feynman diagrams for single-pion production [3] beyond the resonance region, due to the presence of intermediate nucleon resonance states [4]. The charge-odd contribution to the cross-section has a clear signal: the invariant mass of two pions equals the ρ-meson mass and may thus be separated from the even part of the cross-section in a two-meson exclusive set-up.

The idea of measuring the distributions in the fragmentation region of one of the colliding beams was first considered in papers of the seventies [5]. In particular, the odd part of the cross-section for the production of a muon pair in a jet moving along the initial-electron direction was obtained there. The energy distributions in a jet moving along, for example, the initial-electron direction were obtained in the form of a Dalitz-plot for jets consisting of two electrons and one positron and for those of one electron and two photons. For instance, the charge-odd contribution to the spectrum for muon-pair production takes the form

$$\begin{aligned}\frac{\mathrm{d}^2\sigma_{\mathrm{odd}}^{e^+e^-\to e^+e^-\mu^+\mu^-}}{\mathrm{d}x_+\,\mathrm{d}x_-} &= \frac{4\alpha^4}{3\pi m_\mu^2}\ln\left(\frac{s}{m_e m_\mu}\right)\frac{x_+x_-(x_+-x_-)(2-x_+-x_-)}{(x_++x_-)^4} \\ &= 0.58\,\mathrm{nb}\,\cdot\,f_4(x_+,x_-),\end{aligned} \tag{1}$$

with

$$x_{\pm} = \frac{E_{\pm} + p^z_{\pm}}{2E}, \qquad s = 4E^2, \tag{2}$$

where E is the electron beam energy in the centre-of-mass reference frame; $x_{\pm}$ are the muon energy fractions; $E_{\pm}$, $p^z_{\pm}$ the energies and z-components of the muon momenta and m_e and m_μ are the electron and muon masses respectively.

Bearing in mind that the even part of the cross-section has a similar order of magnitude [6], one sees that the asymmetry effects are of order unity. We first consider the jet-2 kinematics, specified by interference of the amplitudes associated with the diagrams depicted in Figs. 1a (double-photon mechanism) and 1b (bremsstrahlung along p_2). The case of the jet-1 kinematics, characterised by interference of the amplitudes graphically represented in Figs. 1a and 1c, is then elaborated in some detail. Then, we derive reduced formulæ, for a back-to-back kinematics with pions moving almost in opposite directions, of the expressions obtained earlier. In conclusion we apply our results to a particular experimental setup (HERA).

2 Jet-2 kinematics

In this paper we deal with the charged-pion pair production channel in high-energy collisions between unpolarised electrons and transversely polarised protons. We first examine the kinematical region for the creation of a jet moving close to initial proton direction:

$$e(p_1) + p(p_2, a) \to e(p_1') + p(p_2') + \pi^+(q_2) + \pi^-(q_1), \tag{3}$$

$$p_1^2 = p_1'^2 = m_e^2, \quad p_2^2 = p_2'^2 = M^2, \quad q_{1,2}^2 = m^2, \quad p_2{\cdot}a = 0,$$

where m_e, M and m are the electron, proton and pion masses respectively and a is the proton polarisation 4-vector. Let us now define the (virtual) exchange-photon 4-momenta in the problem:

$$k_1 = p_1 - p_1', \qquad k_2 = p_2 - p_2',$$

together with

$$k = k_1 + k_2 = q_1 + q_2, \qquad Q = q_1 - q_2.$$

Exploiting the infinite-momentum frame approach, we may neglect the contribution from the diagram of Fig. 1c and thus write down the matrix element in the following form

$$\mathcal{M}_{\text{jet}-2} = \mathcal{M}_D + \mathcal{M}_B = \frac{(4\pi\alpha)^2}{k_1^2}\, \bar{u}(p_1')\gamma_\mu u(p_1)\; \bar{u}(p_2')J^\mu u(p_2). \tag{4}$$

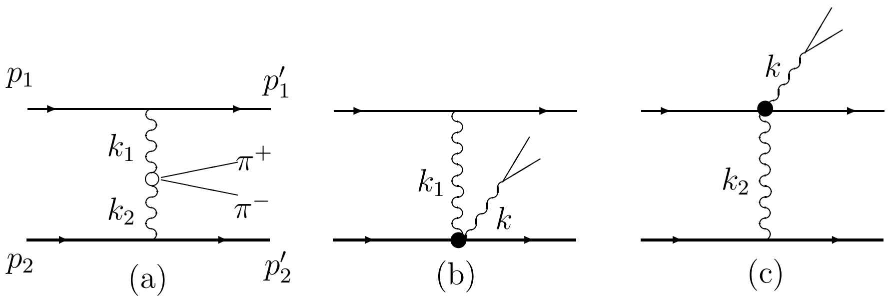

Figure 1: *Diagrams for pion pair production in $e - \vec{p}$ collisions.*

From the double-photon (D) and bremsstrahlung (B) mechanisms for pion pair production, the current receives contributions of the form:

$$J^\mu = J^\mu_D + J^\mu_B, \quad J^\mu_D = \frac{1}{k_2^2}\,\mathcal{M}^{\mu\nu}\gamma_\nu, \quad J^\mu_B = \frac{F(k^2)}{k^2}\,\mathcal{O}^{\mu\nu}Q_\nu, \tag{5}$$

where the pion form factor is taken to be of the form

$$F(k^2) = \frac{m_\rho^2}{k^2 - m_\rho^2 + \mathrm{i}m_\rho\Gamma_\rho}.$$

We have introduced here the following tensors (see Fig. 2 for the graphical correspondence):

$$\mathcal{M}^{\mu\nu} = \frac{(2q_1-k_1)^\mu(k_2-2q_2)^\nu}{\chi_1} + \frac{(2q_1-k_2)^\nu(k_1-2q_2)^\mu}{\chi_2} - 2g^{\mu\nu} \tag{6}$$

(defining $\chi_1 = k_1^2 - 2k_1{\cdot}q_1$ and $\chi_2 = k_1^2 - 2k_1{\cdot}q_2$) and

$$\mathcal{O}^{\mu\nu} = \gamma^\nu\,\frac{\hat{p}_2+\hat{k}_1+M}{(p_2+k_1)^2-M^2}\,\gamma^\mu + \gamma^\mu\,\frac{\hat{p}_2'-\hat{k}_1+M}{(p_2'-k_1)^2-M^2}\,\gamma^\nu, \tag{7}$$

which are subject to the gauge conditions

$$\begin{aligned} \mathcal{M}^{\mu\nu}k_{1\mu} = 0 = \mathcal{M}^{\mu\nu}k_{2\nu}, \\ \bar{u}(p_2')\mathcal{O}^{\mu\nu}u(p_2)\,k_{1\mu} = 0 = \bar{u}(p_2')\mathcal{O}^{\mu\nu}u(p_2)\,k_\nu. \end{aligned} \tag{8}$$

We introduce the standard Sudakov parametrisation of the 4-momenta in the problem

$$\begin{aligned} q_i &= x_i\tilde{p}_2 + \beta_i\tilde{p}_1 + q_{i\perp}, \\ k_1 &= \alpha\tilde{p}_2 + \beta\tilde{p}_1 + k_{1\perp}, \\ p_2' &= x\tilde{p}_2 + \beta_2'\tilde{p}_1 + p_{2\perp}', \end{aligned} \tag{9}$$

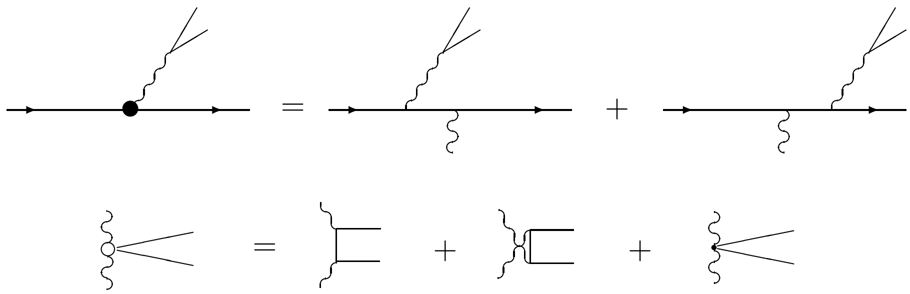

Figure 2: *Decoding of the notation used in Fig. 1.*

where we have used the following:

$$\tilde{p}_2 = p_2 - \frac{M^2}{s} p_1, \quad \tilde{p}_1 = p_1 - \frac{m_e^2}{s} p_2, \quad s = 2p_1 \cdot p_2 \gg M^2.$$

Here

$$x_i = \frac{q_{iz} + E_i}{2E} \qquad \text{and} \qquad x = \frac{p'_{2z} + E'_2}{2E}$$

are the pion and scattered-proton energy fractions ($x + x_1 + x_2 = 1$), $q_{i\perp}$ and $p'_{2\perp}$ are the 4-momenta transverse to the beam axes. The corresponding euclidean 2-vectors are $\mathbf{q}_i$ and $\mathbf{p}'_2$.

The small parameters, β, may be expressed via these vectors:

$$\beta_i = \frac{\mathbf{q}_i^2 + m^2}{s x_i}, \qquad \beta'_2 = \frac{\mathbf{p}_2'^2 + M^2}{s x}. \tag{10}$$

In terms of these variables, the cross-section and the phase volume is

$$\mathrm{d}\sigma = \frac{1}{8s} \sum |\mathcal{M}|^2 \, \mathrm{d}\Gamma,$$

$$\begin{aligned} \mathrm{d}\Gamma &= \frac{\mathrm{d}^3 p'_1 \, \mathrm{d}^3 p'_2 \, \mathrm{d}^3 q_1 \, \mathrm{d}^3 q_2}{(2\pi)^8 2E'_1 2E'_2 2E_1 2E_2} \, \delta^4(p_1 + p_2 - p'_1 - p'_2 - q_1 - q_2) \\ &= \frac{\mathrm{d}^2 \mathbf{q}_1 \, \mathrm{d}^2 \mathbf{q}_2 \, \mathrm{d}^2 \mathbf{k}_1 \, \mathrm{d}x_1 \, \mathrm{d}x_2}{(2\pi)^8 8 s x x_1 x_2}. \end{aligned} \tag{11}$$

The jet-2 kinematics permits us to use the Gribov representation for the exchange-photon Green function, see Eq. (4):

$$\frac{g^{\mu\nu}}{k_1^2} = \frac{1}{k_1^2} \left[g_\perp^{\mu\nu} + \frac{2}{s} (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) \right] \simeq \frac{2}{s k_1^2} p_1^\mu p_2^\nu, \tag{12}$$

and from the gauge condition, $J^\mu k_{1\mu} = J^\mu(\beta p_1 + k_{1\perp})_\mu = 0$, it follows that

$$J^\mu p_{1\mu} = -\frac{s}{s_2} J^\mu k_{1\perp\mu}. \tag{13}$$

Here we have defined $\tilde{s}_2 = (p_2' + q_1 + q_2)^2 = s_2 + M^2$ and denoted the invariant jet-mass squared by

$$s_2 = s\beta = \frac{\mathbf{p}_1'^2 + (1-x)M^2}{x} + \frac{\mathbf{q}_1^2 + m^2}{x_1} + \frac{\mathbf{q}_2^2 + m^2}{x_2}. \tag{14}$$

For the modulus squared of the matrix element summed over spin states and averaged over the azimuthal angle of the virtual photon, we obtain

$$\sum |\mathcal{M}|^2 = -\frac{(4\pi\alpha)^4}{(k_1^2)^2} \frac{4s^2\mathbf{k}_1^2}{s_2^2} \,\mathrm{Tr}[(\hat{p}_2' + M)J_\perp^\mu(\hat{p}_2 + M)(1 - \gamma_5 \hat{a})\tilde{J}_{\mu\perp}]. \tag{15}$$

This expression contains both the charge-even and charge-odd contributions, in addition to which it contains the spin correlation term associated with the proton polarisation vector, a. The cross-section acquires the well-known Weizsäcker-Williams (WW) enhancement factor:

$$\int \frac{\mathrm{d}^2\mathbf{k}_1 \mathbf{k}_1^2}{\pi (k_1^2)^2} = L, \tag{16}$$

where the quantity L stands for a *large* logarithm, whose value depends on the type of the initial particle with momentum p_1. In the case where it is a proton we have

$$\begin{aligned} k_1^2 &= -\left[\mathbf{k}_1^2 + \left(M\frac{s_1}{s}\right)^2\right], \qquad L = L_p = \ln\left(\frac{ms}{Ms_1}\right)^2, \\ s_1 &= \frac{\mathbf{p}_1'^2}{x} + \frac{\mathbf{q}_1^2 + m^2}{x_1} + \frac{\mathbf{q}_2^2 + m^2}{x_2}, \quad M = M_p, \quad m = m_\pi. \end{aligned} \tag{17}$$

For the case where it is an electron we have

$$k_1^2 = -\left[\mathbf{k}_1^2 + \left(m_e\frac{s_2}{s}\right)^2\right], \qquad L = L_e = \ln\left(\frac{ms}{m_e s_2}\right)^2. \tag{18}$$

The main (logarithmic) contribution to the cross-section comes from the region $|\mathbf{k}_1| \ll |\mathbf{q}_i|$, which provides the relation between the transverse components of jet particles: $\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{p}_2' = 0$.

Upon doing a standard matrix element calculus spin-dependent part of the differential cross-section for the jet-2 kinematics is found to be

$$\begin{aligned}\mathrm{d}\sigma^{\mathrm{spin}}_{\mathrm{jet}-2} &= \frac{\alpha^4 L_e}{\pi^3 s_2^3}\frac{\mathrm{Im}F^*(k^2)M}{k_2^2 k^2} \\ &\quad\times [A(\mathbf{q}_1, x_1; \mathbf{q}_2, x_2)(\mathbf{q}_1 \wedge \mathbf{a})_z - A(\mathbf{q}_2, x_2; \mathbf{q}_1, x_1)(\mathbf{q}_2 \wedge \mathbf{a})_z] \\ &\quad\times \frac{\mathrm{d}^2\mathbf{q}_1\, \mathrm{d}^2\mathbf{q}_2\, \mathrm{d}x_1\, \mathrm{d}x_2}{x_1 x_2 x}, \end{aligned} \tag{19}$$

with the expression for A given in the Appendix. Its reduced version for a back-to-back kinematics is quoted below.

3 The general form of $\gamma\gamma \to \pi^+\pi^-$ amplitude

The tensor describing the conversion of two photons into a pion pair can be presented in a form that may be interpreted as an expansion in pion momenta[1]:

$$\mathcal{M}^{\mu\nu} = a_0 \mathcal{L}_0^{\mu\nu} + a_2 \mathcal{L}_2^{\mu\nu}, \tag{20}$$

where

$$a_0 = \frac{\chi_1 + \chi_2}{\chi_1 \chi_2}, \quad a_2 = \frac{2}{\chi_1 \chi_2},$$

$$\begin{aligned}\mathcal{L}_0^{\mu\nu} &= k_1{\cdot}k_2 g^{\mu\nu} - k_1^\nu k_2^\mu, \\ \mathcal{L}_2^{\mu\nu} &= -k_1{\cdot}k_2 Q^\mu Q^\nu - Q{\cdot}k_1(Q^\mu k_1^\nu + Q^\nu k_1^\mu) \\ &\quad + Q{\cdot}k_1 Q^\nu (k_1 + k_2)^\mu + (Q{\cdot}k_1)^2 g^{\mu\nu},\end{aligned}$$

and

$$\mathcal{L}_i^{\mu\nu} k_{1\mu} = 0 = \mathcal{L}_i^{\mu\nu} k_{2\nu}.$$

Taking into account final-state interactions, one expects these amplitudes, as well as the amplitude for the conversion of a single photon into a pion pair, to acquire the following phases:

$$a_0 \to a_0 \mathrm{e}^{\mathrm{i}\delta_0}, \quad F(k^2) \to F(k^2)\mathrm{e}^{\mathrm{i}\delta_1}, \quad a_2 \to a_2 \mathrm{e}^{\mathrm{i}\delta_2}. \tag{21}$$

The phases $\delta_{0,1,2}$ are associated with states having orbital angular momentum equal to $0, 1, 2$ respectively. It is useful to note that the third possible gauge-invariant structure,

$$\mathcal{L}_3^{\mu\nu} = Q{\cdot}k_1(k_1^2 g^{\mu\nu} - k_1^\mu k_1^\nu) + Q^\nu(k_1^2 k_2^\mu - k_1{\cdot}k_2 k_1^\mu), \tag{22}$$

[1]Here a_0 and a_2 may be associated with the partial-wave decomposition of pion–pion scattering amplitudes (not to be confused with the contribution of states with isotopic spin 0 and 2). The quantity a_0 at threshold is connected with the pion polarisability $\alpha_\pi = a_0/(8\pi m)$.

which may be thought of as a pair in a state with unit orbital angular momentum, is not realised in the double-photon channel[2].

4 Jet-1 kinematics

Similar considerations may be applied to the jet-1 kinematics: where a jet moving close to the initial-electron direction consists of a scattered electron and a pion pair. Here we use the following parametrisation of the momenta:

$$\begin{aligned} q_i &= \alpha_i \tilde{p}_2 + x_i \tilde{p}_1 + q_{i\perp}, \\ p_1' &= \alpha_1' \tilde{p}_2 + x \tilde{p}_1 + p_{1\perp}', \\ k_1 &= \alpha \tilde{p}_2 + \beta \tilde{p}_1 + k_{1\perp}, \end{aligned} \tag{23}$$

with

$$\alpha_i = \frac{\mathbf{q}_i^2 + m^2}{s x_i} \qquad \text{and} \qquad \alpha_1' = \frac{\mathbf{p}_1'^2}{s x}, \tag{24}$$

where x and x_i are the energy fractions of a scattered electron and produced pions ($x + x_1 + x_2 = 1$); $\mathbf{p}_1'$ and $\mathbf{q}_i$ are their transverse momenta, obeying $\mathbf{p}_1' + \mathbf{q}_1 + \mathbf{q}_2 = 0$.

We thus obtain

$$\begin{aligned} \sum |\mathcal{M}_D|^2 &= -\frac{16(4\pi\alpha)^4 \mathbf{k}_1^2 s^2}{(k_1^2 k_2^2 s_1)^2} \, \frac{1}{4} \mathrm{Tr} \left[\hat{p}_1' \hat{R}_\perp^\sigma \hat{p}_1 \hat{\tilde{R}}_{\sigma\perp} \right], \\ \sum |\mathcal{M}_B|^2 &= -\frac{16(4\pi\alpha)^4 \mathbf{k}_1^2 |F(k^2)|^2 s^2}{(k_1^2 k^2 s_1)^2} \, \frac{1}{4} \mathrm{Tr} \left[\hat{p}_1' \hat{B}_\perp^\sigma \hat{p}_1 \hat{\tilde{B}}_{\sigma\perp} \right], \\ 2\mathrm{Re} \sum (\mathcal{M}_B \mathcal{M}_D^*) &= -\mathrm{Re} \frac{32(4\pi\alpha)^4 \mathbf{k}_1^2 F(k^2) \mathrm{e}^{\mathrm{i}\delta_1}}{(k_1^2)^2 k_2^2 k^2} \, \frac{1}{4} \mathrm{Tr} \left[\hat{p}_1' \hat{B}_\perp^\sigma \hat{p}_1 \hat{\tilde{R}}_{\sigma\perp} \right], \end{aligned} \tag{25}$$

where

$$s_1 = s\alpha = \frac{\mathbf{p}_1'^2}{x} + \frac{\mathbf{q}_1^2 + m^2}{x_1} + \frac{\mathbf{q}_2^2 + m^2}{x_2}$$

is the jet invariant-mass squared,

$$k = q_1 + q_2, \qquad k_1^2 = -\left[\mathbf{k}_1^2 + M^2 \left(\frac{s_1}{s} \right)^2 \right],$$

and

$$\hat{B}_\sigma = \frac{1}{s} \left(\hat{Q} \hat{p}_2 \gamma_\sigma + \frac{1}{x} \gamma_\sigma \hat{p}_2 \hat{Q} \right) + \frac{2}{s_1 x} p_{1\sigma}' \hat{Q}, \tag{26}$$

[2]We are grateful to V. Serbo and I. Ginzburg for discussion on this point.

and $\hat{R}_\sigma = \mathcal{M}_{\sigma\mu}\gamma^\mu$, see Eq. (6). Proton spin correlations are absent here since in the WW approximation the exchange photon cannot carry spin information.

Taking into account the strong phases in the pion amplitudes, the expression for differential cross-section is modified as follows:

$$\begin{aligned}\mathrm{d}\sigma^{\mathrm{odd}}_{\mathrm{jet-1}} &= -\frac{\alpha^4 L_p}{\pi^3 s_1^2 k_2^2 k^2}\left\{I\,\mathrm{Re}\left[F(k^2)e^{\mathrm{i}(\delta_1-\delta_2)}\right]\right.\\ &\left.+\tilde{I}\,\mathrm{Re}\left[F(k^2)\left(e^{\mathrm{i}(\delta_1-\delta_0)}-e^{\mathrm{i}(\delta_1-\delta_2)}\right)\right]\right\}\frac{\mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2\,\mathrm{d}x_1\,\mathrm{d}x_2}{xx_1x_2}. \qquad (27)\end{aligned}$$

5 Back-to-back kinematics

For the kinematics with $|\mathbf{p}'_1| \to 0$ the charge-odd part of the cross-section takes the form:

$$\begin{aligned}\mathrm{d}\sigma^{\mathrm{odd}}_{\mathrm{jet-1}}\Big|^{(\pi\pi)}_{\mathbf{q}_2=-\mathbf{q}_1} &= \frac{\alpha^4 L_p}{\pi^3}\frac{\mathbf{q}_1^2-\mathbf{q}_2^2}{(\mathbf{q}_1+\mathbf{q}_2)^2}\frac{(x_1x_2)^2}{(\mathbf{q}_1^2+m^2)^3x(1-x)^5} \qquad (28)\\ &\times\left\{-2x\mathrm{Re}\left[F(k^2)\left(\mathrm{e}^{\mathrm{i}(\delta_1-\delta_0)}-\mathrm{e}^{\mathrm{i}(\delta_1-\delta_2)}\right)\right]\right.\\ &\left.+\mathrm{Re}\left[F(k^2)\mathrm{e}^{\mathrm{i}(\delta_1-\delta_2)}\right]\left(1+x^2-\frac{m^2}{\mathbf{q}_1^2+m^2}(1+x)^2\right)\right\}\\ &\times\ \mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2\,\mathrm{d}x_1\,\mathrm{d}x_2,\end{aligned}$$

for $\mathrm{e}^{-\mathrm{i}\delta_i} = 1$, this is in agreement with the expression obtained in [2, 7]. We also include here a similar expression for muon-pair production in this limit:

$$\begin{aligned}\mathrm{d}\sigma^{\mathrm{odd}}_{\mathrm{jet-1}}|^{(\mu\mu)}_{\mathbf{q}_2=-\mathbf{q}_1} &= \frac{\alpha^4 L_p}{\pi^3}\frac{\mathbf{q}_1^2-\mathbf{q}_2^2}{(\mathbf{q}_1+\mathbf{q}_2)^2}\frac{x_1x_2}{(\mathbf{q}_1^2+m^2)^4x(1-x)^5}\\ &\times\left\{\left[(1-x)^2(\mathbf{q}_1^2+m^2)-2\mathbf{q}_1^2x_1x_2\right](1+x^2)+4xx_1x_2m^2\right\}\\ &\times \mathrm{d}^2\mathbf{q}_1\,\mathrm{d}^2\mathbf{q}_2\,\mathrm{d}x_1\,\mathrm{d}x_2. \qquad (29)\end{aligned}$$

For the kinematics where $\mathbf{p}'^2_1 = (\mathbf{q}_1+\mathbf{q}_2)^2 \ll \mathbf{q}_1^2$ we have (using the δ-function approximation for $\mathrm{Im}F^*$):

$$\begin{aligned}\mathrm{d}\sigma^{\mathrm{spin}}_{\mathrm{jet-2}}|^{(\pi\pi)}_{\mathbf{q}_2=-\mathbf{q}_1} &= \frac{2\alpha^4 L_e}{\pi^2}M(\mathbf{q}_1\wedge\mathbf{a})_z\\ &\times\frac{(xx_1x_2)^2}{v(M^2(1-x)^2+\mathbf{p}'^2_2)(1-x)^3(vx+M^2x_1x_2)^3}\\ &\times\left[v-m^2(1+x)-\frac{M^2x_1x_2}{x}-\frac{v}{4}\frac{(1-x)^3}{x_1x_2}\right] \qquad (30)\end{aligned}$$

$$\times \, \delta \left(1 - \frac{v(1-x)^2}{x_1 x_2 m_\rho^2} \right) \, \mathrm{d}^2\mathbf{q}_1 \, \mathrm{d}^2\mathbf{q}_2 \, \mathrm{d}x_1 \, \mathrm{d}x_2,$$

where $v = \mathbf{q}_1^2 + m^2$.

Let us now discuss the accuracy of the formulæ presented, determined, of course, by the omitted terms: they are of order

$$\frac{m^2}{s}, \quad \frac{s_i}{s}, \quad \text{and} \quad \frac{1}{L}, \tag{31}$$

as compared to unity. For the DESY experimental conditions the accuracy is better than 10%. For the inclusive set-up, one must consider the 3-pion production process. Compared with 2-pion production, this has a phase-volume suppression factor

$$\int \frac{\mathrm{d}^2\mathbf{q}}{\pi M^2} \sim \left(\frac{m}{M} \right)^2 \sim 10^{-2}.$$

For intermediate energies (such as at VEPP-2M and DAΦNE), despite the rather small suppression factor $m^2/s \sim 0.01$ for the theoretical background of the remaining Feynman diagrams (there are six gauge-invariant sets, of which we have considered just two), the WW enhancement factor (a large logarithm) and the specific choice of kinematics ($|\mathbf{q}_1 + \mathbf{q}_2| \sim 0$) provides an accuracy of the same order for jet-1 kinematics as obtained for jet-2 type.

6 Conclusion

Using conditions of the back-to-back kinematics as well as the ρ meson dominance approximation the charge-odd contribution to the spectrum in a jet-1 kinematics for the case $|\mathbf{p}_1'| \ll |\mathbf{q}_1|$ may be cast into the following form:

$$\begin{aligned} m_\rho^2 \frac{\mathrm{d}\sigma_{\mathrm{jet}-1}^{\mathrm{odd}(\pi^+\pi^-)}}{\mathrm{d}^2\mathbf{q}_1 \, \mathrm{d}x_1 \, \mathrm{d}x_2} &= 0.7 \cdot 10^{-2} \, \mathrm{nb} \\ &\times \frac{\mathbf{q}_1^2 - \mathbf{q}_2^2}{(\mathbf{q}_1 + \mathbf{q}_2)^2} \, [f_1 \sin(\delta_0 - \delta_1) + f_2 \sin(\delta_2 - \delta_1)], \end{aligned} \tag{32}$$

with f_1 and f_2 being smooth functions of order unity:

$$f_1 = -\frac{2}{x_1 + x_2}, \qquad f_2 = \frac{(1+x)^2}{x(x_1 + x_2)}. \tag{33}$$

The contribution to the spin-dependent part in the same kinematics can be cast into the form

$$\frac{\mathrm{d}\sigma_{\mathrm{jet}-2}^{\mathrm{spin}(\pi^+\pi^-)}}{\mathrm{d}\phi \, \mathrm{d}x_1 \, \mathrm{d}x_2} = 1.8 \cdot 10^{-2} \, \mathrm{nb} \, \cdot |a| f_3 \frac{\mathrm{d}\mathbf{p}_1'^2}{M^2(1-x)^2 + \mathbf{p}_1'^2} \cdot \sin\phi,$$

x_1	x_2	f_1	f_2	f_3	f_4
0.2	0.2	-5.000	10.667	0.030	0.000
0.2	0.4	-3.333	8.167	-0.068	-0.173
0.2	0.6	-2.500	9.000	-0.053	-0.141
0.3	0.4	-2.857	8.048	-0.075	-0.065

Table 1: *The values of the functions* f_1, f_2 [Eq. (33)], f_3 [Eq. (34)] *and* f_4 [Eq. (1)] *for typical HERMES conditions* ($|\mathbf{p}_1'| \ll |\mathbf{q}_1|$, $0.2\,\mathrm{GeV} < |\mathbf{q}_1| < 1.2\,\mathrm{GeV}$, $0.2 < x_i < 0.6$).

with

$$f_3 = \frac{1.22x}{\sqrt{x_1 x_2}} \frac{[x_1 x_2 x - 1.49(x_1+x_2)^2 x_1 x_2 - 0.25x(x_1+x_2)^3]}{[x + 1.49(x_1+x_2)^2]^3}. \tag{34}$$

Here $|a|$ is the degree of target transverse polarisation; ϕ is the azimuthal angle between the directions of target polarisation and transverse momentum, $\mathbf{q}_1$, of a negative pion. Typical values of the functions f_i for a set of x_1, x_2 values are given in Table 1 for HERMES conditions ($s = 60\,\mathrm{GeV}^2$ and $|\mathbf{q}_1| \approx |\mathbf{q}_2| \sim 0.5 \div 1.5\,\mathrm{GeV}$).

The calculation presented above has been carried out within the framework of QED. In the case of jet-1 kinematics it is possible to replace photon exchange between the pion and nucleon by that of a *pomeron* in the double-photon amplitudes. This results in an enhancement factor α_s/α: $\sigma_0 \to (\alpha_s/\alpha)\sigma_0$.

A visible effect of the asymmetry in pion pair production was measured at Cornell[3] where the number of most energetic π^- moving along the e^- direction exceeds that of most energetic π^+ in the same direction.

The formulæ given above may be applied to pion pair production at e^+e^- colliders. Besides the effect of contributions of the annihilation type, Feynman diagrams fall with the growth of the total centre-of-mass energy ($\sqrt{s}$) since they are of order $m^2/s < 3\%$ for J/ψ and B factories. Nevertheless, charge-odd effects in $\pi^+\pi^-$ production may definitely be measured at Φ-factories by taking advantage of the kinematics discussed above. Moreover, charge-odd effects in K_S and K_L pair production may clearly be seen at J/ψ and B factories.

[3]We thank V.G. Serbo for bringing this to our attention.

Appendix

The general expression in a jet-1 kinematics for the charge-odd contribution to a pion pair production cross-section in the WW approximation is given in Eq. (27) with the quantities I and $\tilde{I}$:

$$
\begin{aligned}
I \quad = \quad & -\mathbf{Q}\mathbf{p}_1' + \frac{(1+x)(x_1-x_2)}{x} p_1 \cdot p_1' + \frac{1+x}{2x_1x_2}\mathbf{b}\mathbf{Q} - \frac{x_1-x_2}{2x_1x_2}\mathbf{b}\mathbf{p}_1' \\
& - \frac{\mathbf{p}_1'\mathbf{a}}{x_1x_2x} \frac{(2p_1 \cdot Q 2p_1' \cdot Q - Q^2 2p_1 \cdot p_1')}{s_1^2} + \frac{2(-\mathbf{p}_1'^2 Q \cdot p_1 + \mathbf{Q}\mathbf{p}_1' p_1 \cdot p_1')}{xs_1} \\
& - \frac{\mathbf{Q}\mathbf{a}}{x_1x_2s_1}(p_1 \cdot Q + p_1' \cdot Q) + \frac{\mathbf{p}_1'\mathbf{b}}{x_1x_2s_1}(xQ \cdot p_1 + Q \cdot p_1' - (x_1-x_2)p_1 \cdot p_1') \\
& - \frac{(1-x)Q^2}{2s_1} \frac{\mathbf{p}_1'\mathbf{a}}{x_1x_2x} + \frac{x_1-x_2}{xx_1x_2s_1} Q \cdot p_1 \mathbf{p}_1'\mathbf{a},
\end{aligned} \tag{35}
$$

where

$$
\begin{gathered}
\mathbf{b} = x_2\mathbf{q_1} + x_1\mathbf{q_2}, \qquad \mathbf{a} = x_2\mathbf{q_1} - x_1\mathbf{q_2}, \\
k_2^2 = -2p_1 \cdot p_1' = -\frac{\mathbf{p}_1^{2\prime}}{x}, \qquad 2p_1' \cdot Q = x2p_1 \cdot Q - 2\mathbf{Q}\mathbf{p}_1', \\
2p_1 \cdot Q = \frac{1}{x_1}(\mathbf{q}_1^2 + m^2) - \frac{1}{x_2}(\mathbf{q}_2^2 + m^2) \\
Q^2 = 4m^2 - k^2, \qquad k^2 = \frac{1}{x_1x_2}\left(m^2(1-x)^2 + \mathbf{a}^2\right),
\end{gathered} \tag{36}
$$

and

$$
\begin{aligned}
\tilde{I} \quad = \quad & -\frac{2}{1-x}\mathbf{p}_1'\mathbf{Q} + \mathbf{p}_1'^2 \left[\frac{(1-x+x^2)(x_1-x_2)}{x^2(1-x)} - \frac{4p_1 \cdot Q}{s_1 x(1-x)} \right] \\
& + \frac{1+x}{x^2(1-x)} \frac{\mathbf{p}_1'^2\, \mathbf{p}_1'\mathbf{Q}}{s_1} + \frac{x_1-x_2}{x^2(1-x)} \frac{(\mathbf{p}_1'^2)^2}{s_1}.
\end{aligned} \tag{37}
$$

The quantity A, relevant for the spin asymmetry, is

$$
\begin{aligned}
\mathcal{A}(\mathbf{q}_1, x_1; \mathbf{q}_2, x_2) \quad = \quad & -s_2\frac{1-x}{x} + \frac{(1-x)^2Q^2}{2xx_1} + \frac{x_1-x_2}{2xx_1}(-2xp_2' \cdot Q + 2p_2 \cdot Q) \\
& + \frac{2}{xx_1}[x_2\mathbf{q}_1^2 + x_1\mathbf{q}_2^2 - (1-x)\mathbf{q}_1\mathbf{q}_2] - \frac{2x_2}{xx_1}(\mathbf{q}_1^2 - \mathbf{q}_2^2) \\
& + \left[\frac{x_1}{x_2}(\mathbf{q}_2^2 + m^2) - \frac{x_2}{x_1}(\mathbf{q}_1^2 + m^2)\right]\left[\frac{x_1-x_2}{xx_1} + \frac{2}{s_2xx_1}(\mathbf{q}_2^2 - \mathbf{q}_1^2)\right].
\end{aligned} \tag{38}
$$

References

[1] A. Belkov and S. Bunyatov, *Phys. of El. Part. and At.* **13**, Part 1 (1982); A. Belkov, S. Bunyatov, K. Mukhin and O. Patarakin, *Pion-pion interaction*, “Energoatomizdat” (1985) (In Russian); L. Nemenov, *Sov. J. Nucl. Phys.* **41** (1985) 629.

[2] V. Serbo and V. Chernyak, *Yad. Fiz.* **19** (1974) 123.

[3] A. Ahmedov, I.V. Akushevich, É.A. Kuraev and P.G. Ratcliffe, e-print hep-ph/9902418, *Eur. Phys. J.* **C11** (1999) 703-708.

[4] A. Arbuzov *et al.*, *Phys. Atom. Nucl.* **57** (1994) 1004.

[5] É.A. Kuraev, L.N. Lipatov, N. Merenkov and M. Strikman, *Yad. Fiz.* **23** (1976) 163.

[6] V. Baier, V. Fadin, V. Khoze and É.A. Kuraev, *Phys. Rep.* **78** (1981) 293.

[7] V. Budnev, I. Ginzburg, G. Meledin and V. Serbo, *Phys. Rep.* **15** (1975) 181.